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Evaluation of elastic modulus for unidirectionally aligned short fiber composites †

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Abstract

An analytical approach of to reinforcement for of short fiber reinforced composites has been extended to include the estimation of elastic modulus. The model is based on the theoretical development of shear lag theory developed by Cox for unidirectionally Aligned aligned Short short Fiber fiber Compositescomposites. Thus, the evolution of conventional models is described in detail along with the effect on the modulus of various parameters. Results are shown with experimental data as well as the comparison of other theories. It is found that the present model agrees well with experimental data and resolves some of the discrepancies among the previous models. It is also found that the present model is very accurate yet relatively simple to predict Young's modulus of discontinuous composites and has the capability to correctly predict the effects of fiber aspect ratio, fiber volume fraction, and fiber/matrix modulus ratio.

Keywords: Piezoelectric hollow cylinder; Thermal gradient; Electric potential; Radial stress; Hoop stress

1. Introduction

Composite materials are among the strongest candidates as a structural material for many automobile, aerospace and other applications [1, 2]. Among them, short fiber reinforced composites or discontinuous composites are not as strong or as stiff as continuous fiber reinforced composites and are not likely to be used in critical structural applications such as aircraft primary structures. However, they do have several attractive characteristics that make them worthy of consideration for other applications. Therefore, short fiber reinforced composite materials have been extensively investigated because they are more economical and impact resistant [3].

One of the earliest attempts to explain the reinforcing effect of short fibers was described by Cox [4], and is now referred to as the shear lag theory which considers long straight discontinuous fibers completely embedded in a continuous matrix [1, 4]. It is the most conventional version of the shear model which can be considered as debonded at fiber ends, assuming that no stress is transferred across the fiber ends. However, the prediction of composite modulus calculated by Cox model does not provide sufficiently accurate strengthening predictions when the fiber aspect ratio is small [5, 6]. The predicted modulus value obtained by Cox model is significantly smaller than the experimentally observed values in the short fiber composites. In fact, the Cox model gives an underestimation of the strength due to the neglect of stress transfer across the fiber ends [3, 7, 8].

Over the years, several ways of accounting fiber end stresses have been proposed. Nardone and Prewo attempted to modify the conventional shear lag model to take into account the tensile transfer of load from the matrix to the discontinuous reinforcement [5]. Their modified shear lag model is well fitted with the experimental data for prediction of yield strength of discontinuous composites. However, it is limited to pre-

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dict the fiber stress with constant value after matrix yielding, which does not have the capability to calculate an estimation of elastic behavior. Subsequently, Taya and Arsenault considered reinforced fiber end stress as the average stress of remote matrix though it has the limitation for no stress intensification between fibers [9]. Their results also show an underestimation for the prediction of elastic modulus in short fiber reinforced composites.

There have been more attempts such as the Clyne model considering that the fiber end stress is assumed to the mean value between the average value in the matrix remote from the interface and the peak value in the fiber in the absence of fiber end stresses [3]. It has been shown that the Clyne model is an improvement over the Cox model or Taya model in terms of the prediction of Young's modulus data of MMCs (metal matrix composites) such as SiC-Al composites.

Recently, Starink also proposed within the framework of shear lag model, assuming that the fiber end stress would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the presence of fiber end stresses. However, both models are not based on the material properties so that they only postulate an average value between the fiber maximum stress and the fiber end stress [8]. More recently, Kim reassessed the fiber end stress accounting for stress concentration phenomenon for the accurate prediction of reinforcement effects based on the material property [7, 10].

Consequently, in this paper, an analytical approach to reinforcement effects in short fiber reinforced composites has been extended to include the estimation of elastic modulus. The predicted values of elastic modulus calculated by the present model show very accurate results with experimental data though the formulation is relatively simple. It is found that the present model gives a reasonable closed form solution and has the capability to correctly predict the value of elastic Young's modulus as well as the effects of fiber aspect ratio, fiber volume fraction, and fiber/matrix modulus ratio.

2. Analysis

Formulations for the longitudinal modulus of the aligned discontinuous fiber composite can be derived by using the shear lag approach from Cox [4]. The short fibers are considered to be uniaxially aligned with the stress applied in the axial direction of the fibers as described in Fig. 1. It is considered a composite containing fibers which all have the same length and diameter, and are all parallel. Hence, any section normal to the bar axis MN for instance, will intersect fibers at all possible positions along their length, so long as there is a large number of them in the cross section. The load carried by the fibers will be the total fiber area across the bar section, multiplied by the average fiber stress. It is presumed that the area fraction is equal to the volume fraction in any random cross section. For a thorough understanding of the shear lag model in discontinuous composites, it is necessary to first understand the mechanism of stress transfer. Hence, a micro-mechanical model of short fiber reinforced composites can be selected as an RVE (representative volume element) shown in Figs. 2 and 3.

Fig. 2 depicts the composite unit cell showing the short fiber embedded in a continuous matrix. The outer surface of the unit cell can be considered as having a hexagonal contour. However, the exact shape is not critical, so that the unit cell is treated as an equivalent cylinder. Fig. 3 shows that the short fibers are aligned with the stress applied in the axial direction (z-axis) of the fibers. Further, no plastic yielding is allowed, that is, both matrix and fiber de-

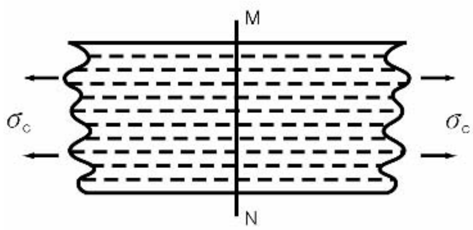


Fig. 1. Schematic diagram of short fiber reinforced composite with far field composite stress σ_c . MN represents a random cross-section in an aligned fiber composite.

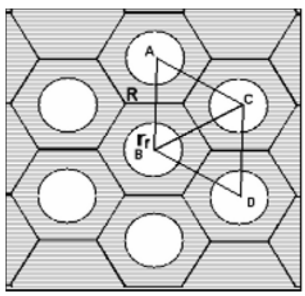


Fig. 2. An axisymmetric RVE having fiber radius r_f and cell radius R : Regularly arranged hexagonal model.

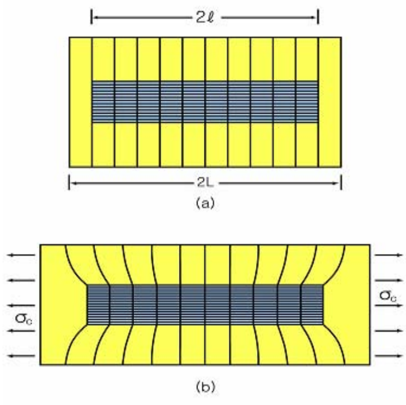


Fig. 3. RVE for aligned discontinuous fiber composite with an elastic fiber and matrix. (a) Unstrained RVE before deformation, (b) Strained RVE after deformation.

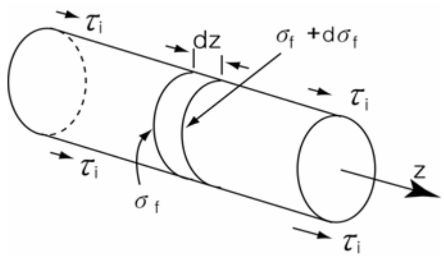


Fig. 4. Free body diagram of the fiber, which indicates fiber axial stress and shear stress at the fiber surface, respectively.

form in a purely elastic manner.

As shown by the grid lines before and after deformation in Fig. 3, the stiffness mismatch between fiber and matrix (usually E_f>>E_m) leads to large shear deformations near the fiber ends but no shear deformation at the middle of the fiber. Hence, E_m and E_f are Young's moduli of the matrix and the fiber, respectively. The diameter of fiber and RVE is r_f and R. Likewise, the shear stress of fiber surface and arbitrary surrounding matrix in the z-direction is τ_i and τ, respectively. For convenience, the fiber radius is fixed as unit length, i.e,, r_f=1, so that the aspect ratio, s, has the same length of the normalized fiber distance.

In Fig. 4, shear forces at arbitrary distance with r and those at the fiber surface r_f in the composite element are

$$2\pi r\tau dz = 2\pi r_f \tau_i dz \quad \text{or} \quad r = r_f \tau_i / r \,, \quad r_f \le r \le R \tag{1}$$

Using Eq. (1) with simple equilibrium conditions as reported in the previous paper [7], the governing equation is obtained as Eq. (2).

$$\frac{d^2\sigma_f}{dz^2} = \frac{n^2}{r_f^2}(\sigma_f - E_f \varepsilon_c) \tag{2}$$

where ε is the far-field composite strain and n is the dimensionless parameter in associated with Eq. (3).

$$n^2 = \frac{2E_m}{E_f(1+\nu_m)\ln(P_f / V_f)} \tag{3}$$

Here, ν_m is Poison's ratio of the matrix, s (=l/r_f) is fiber aspect ratio, and Pf is packing factor having the value $P_f = 2\pi/\sqrt{3}$ for hexagonal arrangement of fiber. V_f and V_m are volume fractions of the fiber and the matrix, respectively. Then, above equation admits the solution as shown below. Eq. (2) has the solution of Eq. (4).

$$\sigma_f = E_f \varepsilon_c + A\sinh(\frac{nz}{r_f}) + B\cosh(\frac{nz}{r_f}) \tag{4}$$

Hence, constants A and B are dependent on the assumed stress at the fiber ends, σ_e. Taking the origin at the middle of the fiber and accounting for the symmetry gives:

$$\sigma_f = E_f \varepsilon_c \left\{ 1 + (\sigma_e - 1)\frac{\cosh(nz/r_f)}{\cosh(ns)} \right\} \tag{5}$$

Hence, it follows that the axial stress at z=0 is given by:

$$\sigma_f = E_f \varepsilon_c \{1 + (\sigma_e - 1)\mathrm{sec}h(ns)\} \tag{6}$$

In short fiber composites, loads are not directly applied on the fibers but are applied to the matrix and transferred to the fibers through the fiber ends as well as through the cylindrical surface of the fiber. When the length of a fiber is much greater than the length over which the transfer of stress takes place, the end effects can be neglected. However, the end effects significantly influence the behavior of a short fiber reinforced discontinuous composite.

During the past few decades, several ways of determining σ_e have been proposed [4, 7, 8, 9]. In the most conventional version of the shear model which can be considered as debonded at fiber ends, it is assumed that no stress is transferred across the fiber ends as below [4].

$$\sigma_f \equiv \sigma_e = 0 \quad \text{for} \quad z = \pm l \tag{7}$$

This model is not reasonable for well bonded fibers with relatively low fiber aspect ratios, but is fairly adequate for fibers with high fiber aspect ratio. Thus, Eq. (6) yields to Eq. (8), called the Cox model.

$$\sigma_f^{Cox} = E_f \varepsilon_c \left\{ 1 - \frac{\cosh(nz/r_f)}{\cosh(ns)} \right\} \tag{8}$$

On the other hand, the total load carried by the composite can be calculated by using the "Rule of Averages." Thus, the composite stress σ_c can be determined by Eq. (9).

$$\sigma_c = V_f \overline{\sigma_f} + V_m \overline{\sigma_m} \tag{9}$$

Then, the average fiber stress $\overline{\sigma_f}$ can be obtained by integrating Eq. (7) and it results in Eq. (10).

$$\overline{\sigma_f}^{Cox} = \frac{1}{L}\int_0^L \sigma_f^{Cox} dz = \frac{E_f \varepsilon_c}{L}\int_0^L \left\{ 1 - \frac{\cosh(nz/r)}{\cosh(ns)} \right\} dz$$

$$= E_f \varepsilon_c \left(1 - \frac{\tanh(ns)}{ns} \right) \tag{10}$$

where is ε_c the composite strain. By assuming the average matrix stress $\overline{\sigma_m} = E_m \varepsilon_c$, Eq. (8) can be expressed as below.

$$\sigma_c^{Cox} = \left\{ V_f E_f \left(1 - \frac{\tanh(ns)}{ns} \right) + V_m E_m \right\} \varepsilon_c \tag{11}$$

From this, the average longitudinal composite stress σ_c can be calculated and the corresponding composite modulus according to the Cox model E_c^{Cox} is obtained as below.

$$E_c^{Cox} = V_f E_f \left(1 - \frac{\tanh(ns)}{ns} \right) + V_m E_m \tag{12}$$

In the meantime, Taya and Arsenault [9] reported the fiber axial stress by taking fiber end stress into account without stress concentration effect. Accordingly, fiber end stress is imposed by the far field average matrix stress as σ_c in Eq. (5), which is expressed as Eq. (13).

$$\sigma_f \equiv \sigma_c = E^{Taya} \varepsilon_c \quad \text{for} \quad z = \pm l \tag{13}$$

where $E^{taya} = E_m$. Therefore, the fiber axial stress in their equation can be given by Eq. (14).

$$\sigma_f^{Taya} = E_f \varepsilon_c \left\{ 1 + \left(\frac{E_m}{E_f} - 1 \right) \frac{\cosh(nz/r_f)}{\cosh(ns)} \right\} \tag{14}$$

Calculating as the same procedure from Eq. (9) through Eq. (12) after imposing Eq. (5), the composite modulus E^{taya} is obtained as below.

$$E_c^{Taya} = V_f E_f \left\{ 1 + \left(\frac{E^{Taya}}{E_f} - 1 \right) \frac{\tanh(ns)}{(ns)} \right\} + V_m E_m$$

$$= V_f E_f \left\{ 1 + \left(\frac{E_m}{E_f} - 1 \right) \frac{\tanh(ns)}{(ns)} \right\} + V_m E_m \tag{15}$$

On the other hand, Clyne postulated that σ_c would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the absence of fiber end stresses [3]. It can be shown that this leads to:

$$\sigma_f \equiv \sigma_e = E^{Clyne} \varepsilon_c \quad \text{for} \quad z = \pm l \tag{16}$$

where E^{Clyne} is denoted as below.

$$E^{Clyne} = \frac{1}{2}[E_f\{1 - \operatorname{sec}h(ns)\} + E_m] \tag{17}$$

From this, the average longitudinal composite stress σ_c can be calculated and the corresponding composite modulus according to the Clyne model E_c^{Clyne} is obtained as below.

$$E_c^{Clyne} = V_f E_f \left\{ 1 + \left(\frac{E^{Clyne}}{E_f} - 1 \right) \frac{\tanh(ns)}{ns} \right\} + V_m E_m \tag{18}$$

It has been shown that the Clyne model is an improvement over the Cox model in terms of the prediction of Young's modulus data of SiC-Al composites [8]. However, both versions underestimate Young's modulus of such composites [3].

Recently, Starink proposed within the framework of shear lag model, which would approximate to the mean between the average value in the matrix remote from the interface and the peak value in the fiber in the presence of fiber end stresses [14]. It can be shown that this leads to:

$$\sigma_f \equiv \sigma_e = \frac{1}{2}\{E_f\varepsilon_c + (\sigma_e - E_f\varepsilon_c)\sec h(ns) + E_m\varepsilon_c\}$$
$$\text{for } z = \pm l \tag{19}$$

which leads again to Eq. (20)

$$\sigma_f \equiv \sigma_e = E^{Starink}\varepsilon_c \quad \text{for } z = \pm l \tag{20}$$

where $E^{Starink}$ is defined as below.

$$E^{Starink} = \frac{E_f\{1 - \sec h(ns)\} + E_m}{2 - \sec h(ns)} \tag{21}$$

From this, the average longitudinal composite stress σ_c can be calculated and the corresponding composite modulus according to the Starink model $E_c^{Starink}$ is obtained as below [14].

$$E_c^{Starink} = V_f E_f\left[1 + \left(\frac{E_m / E_f - 1}{2 - \sec h(ns)}\right)\frac{\tanh(ns)}{ns}\right] + V_m E_m \tag{22}$$

More Recently, Kim reported a closed form solution of composite mechanics to investigate the fiber elastic-matrix plastic behavior [7] and the fiber elastic-matrix elastic behavior [10]. It was performed in order to predict fiber stresses, fiber/matrix interfacial shear stresses and matrix yielding behavior for short fiber reinforced metal matrix composites. In his framework of shear lag model, σ_e was rigorously estimated by using material property instead of postulation of average concept. The fiber end stresses were evaluated by the stress concentration factor α_k , which is accounted for by the value as a function of modulus ratio given by Eq. (23).

$$\alpha_k = \sqrt{\frac{E_f}{E_m}} \tag{23}$$

From this, it can be shown that this leads to:

$$\sigma_f \equiv \sigma_e = E^{Kim}\varepsilon_c \quad \text{for } z = \pm l \tag{24}$$

where E^{Kim} is denoted as below.

$$E^{Kim} = \sqrt{E_m E_f} \tag{25}$$

Consequently, the average longitudinal composite stress σ_c can be calculated and the corresponding composite modulus according to the Kim model E_c^{Kim} is obtained as below.

$$E_c^{Kim} = V_f E_f\left\{1 + \left(\sqrt{\frac{E_m}{E_f}} - 1\right)\frac{\tanh(ns)}{ns}\right\} + V_m E_m \tag{26}$$

Jiang et al. have also proposed a shear lag type model based on an expression for which differs from the ones presented above [10]. It is further noted that the semi-empirical Halpin-Tsai equation has also been used to describe the elastic response of composite systems [11, 12, 13]. This model is based on the self-consistent method developed by Hill [12]. Hermans employed a self-consistent model to obtain a solution in terms of Hill's "reduced moduli" [13]. Halpin and Tsai reduced Hermans' solution to a simpler analytical work and extended its use for a variety of filament geometries [13]. In the Halpin-Tsai model, some interpolation procedures have been used for design purposes, and it has extensively been used due to its simplicity. These equations for normalized longitudinal and transverse moduli, E_{cl}/E_m and E_{ct}/E_m can be written as Eqs. (27) and (28), respectively.

$$\frac{E_{cl}}{E_m} = \frac{1 + 4\left(\frac{l}{r_f}\right)\eta_l V_f}{1 - \eta_l V_f} \tag{27}$$

$$\frac{E_{ct}}{E_m} = \frac{1 + 2\eta_t V_f}{1 - \eta_t V_f} \tag{28}$$

where

$$\eta_l = \frac{(\frac{E_f}{E_m}) - 1}{(\frac{E_f}{E_m}) + 4(\frac{l}{r_f})} \tag{29}$$

$$\eta_t = \frac{(\frac{E_f}{E_m}) - 1}{(\frac{E_f}{E_m}) + 2} \tag{30}$$

The average modulus for a randomly oriented composite is given by:

$$E_{cr} = \frac{3}{8}E_{cl} + \frac{5}{8}E_{ct} \quad (31)$$

Skibo, for example, has shown that the computed modulus values obtained by using Eq. (24) are in reasonable agreement with experimental modulus values [15]. However, this model is not rigorous enough to assess the overall elastic constants, though it holds a simplicity in application.

On the other hand, there are several models for the prediction. One of them is the "Rule of Mixture (ROM)" model, which is given as Eq. (32). It obviously shows an overestimated prediction for discontinuous composites since it is exactly the case of continuous composites.

$$E_c^{ROM} = V_f E_f + V_m E_m \quad (32)$$

3. Results and discussion

The results of the present study (Kim model) are compared with experimental data as well as the prediction of other models for various fiber aspect ratios, fiber volume fractions, and fiber/matrix Young's modulus ratios. For the numerical calculation, typical elastic moduli of materials are chosen as E_f=450GPa for the fiber and E_m=71GPa and 78GPa for the matrix. The fiber volume fraction V_f is imposed with several cases up to V_f=50% as reality. It should be noted that E_m depends on the compositions of the *Al*-based alloys, especially the *Li* content. In addition, E_m is taken as the average value for the alloys considered in order to make valid comparisons for cases in which E_m varies considerably between the alloys.

Fig. 5 shows the calculated values of dimensionless parameter *n* as a function of fiber volume fraction in case of hexagonal arrangement of fiber as shown in Fig. 2. E_m =71GPa and 78GPa are implemented for Young's modulus of the matrix, and E_f =450GPa is implemented for Young's modulus of the fiber. It shows a slight difference for two different Young's modulus of the matrix.

Fig. 6 shows the predicted and measured composite/matrix Young's modulus ratio as a function of fiber volume fraction for SiC-Al composites with various short fiber aspect ratios such as s=2, 3, and 4.

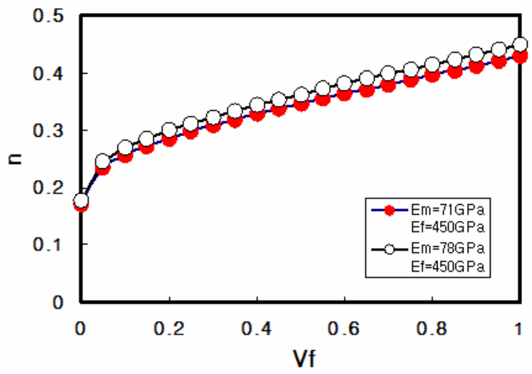
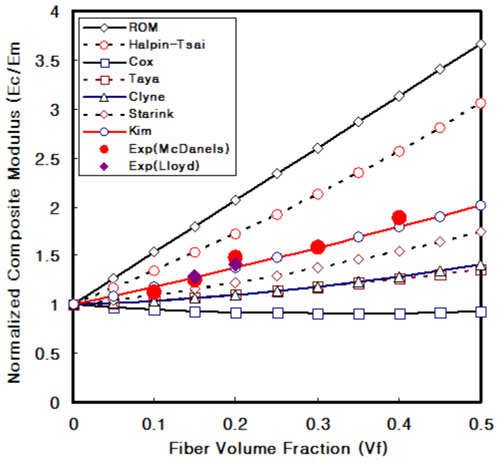


Fig. 5. Calculated values of dimensionless parameter n as a function of fiber volume fraction for hexagonal arrangement of fiber.

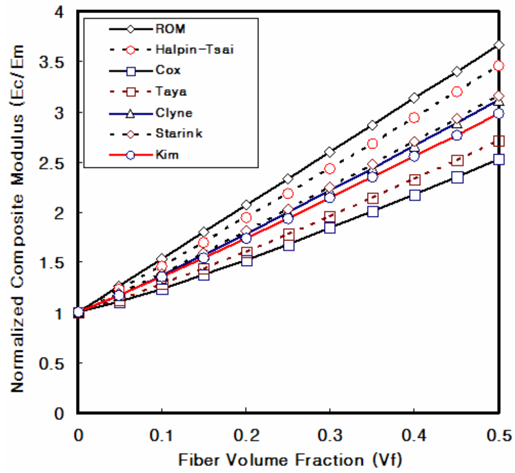
In Fig. 6 (a) in relation to a very small fiber aspect ratio (s=2), it is found that thte ROM model and Halpin-Tsai model overestimate, whereas the Cox model, Taya model, Clyne model, and Starink model underestimate from experimental data. However, it can be seen in Figs. 6 (b) and 6 (c) that the Clyne model, Starink model, and Kim model fit closely along with experimental data. In a very small fiber aspect ratio regime such as s=2, the error percentage of each model shows significantly. The error percentage of the Halpin-Tsai model, indicating the upper bound except ROM model, is 36.3%, that of the present model is 4.9%, and that of the Cox model indicating the lower bound is 51.8%. Note that all the experimental data used in Figs. 6-9 are based on the previous data in the reference [5, 16-20].

Fig. 7 shows the prediction of composite/matrix Young's modulus ratio as a function of fiber volume fraction for SiC-Al composites with the larger fiber aspect ratios (s=8, 16, 32). In Fig. 7 (a), three groups are found: (1) ROM model and Halpin-Tsai model: overestimated group, (2) Cox model and Taya model: underestimated group, (3) Clyne model, Starink model, and Kim model: moderate group. However, it can be seen in Figs. 7 (b) and 6 (c) that all seven models go similarly at the relatively larger fiber aspect ratios (s=16 and 32) as they should.

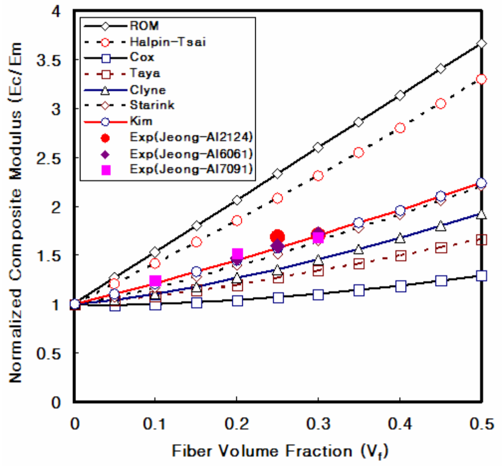
On the other hand, Fig. 8 shows the predicted and measured composite/matrix Young's modulus ratio as a function of fiber aspect ratio for V_f=30% in SiC-Al composites. In Fig. 8, it is found that the ROM model presumably fits a continuous fiber composite and that the Halpin-Tsai model overestimates from experimental data, whereas the Cox model and the Taya



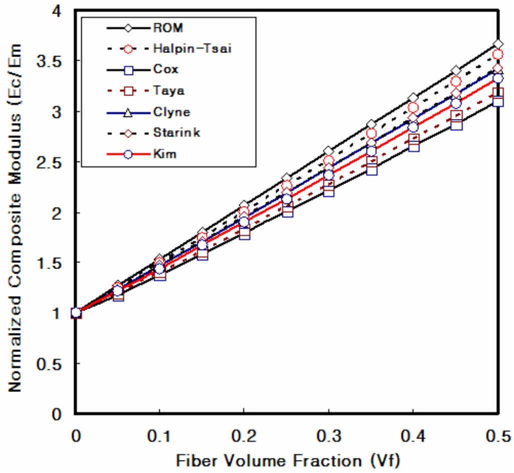
(a)s=2



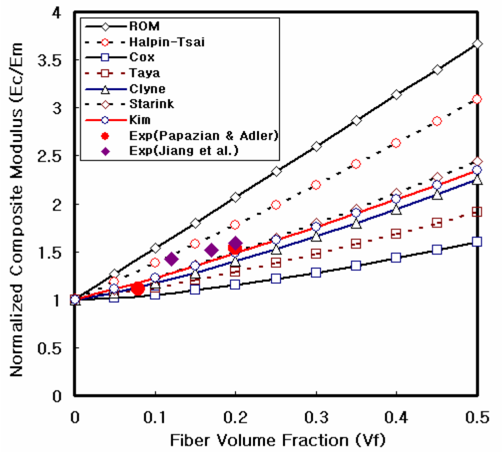
(a) s=8



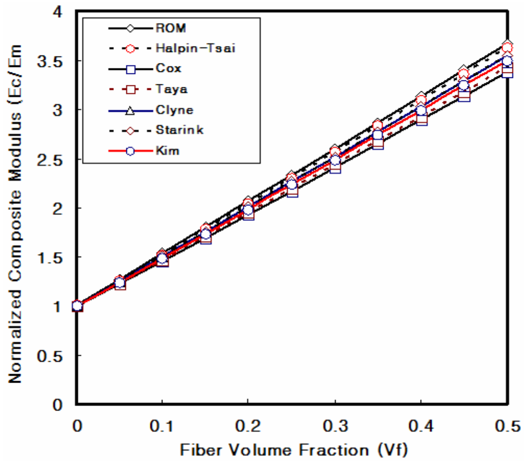
(b) s=3



(b) s=16



(c) s=4



(c) s=32

Fig. 6. Predicted and measured composite/matrix Young's modulus ratio as a function of fiber volume fraction for SiC-Al composites with various short fiber aspect ratios (s=2, 3, 4).

Fig. 7. Predicted composite/matrix Young's modulus ratio as a function of fiber volume fraction for SiC-Al composites with various fiber aspect ratios (s=8, 16, 32).

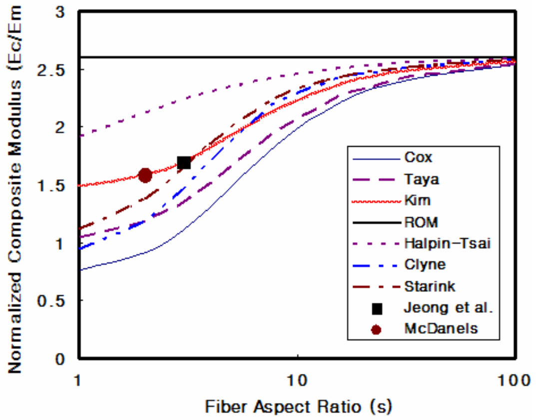


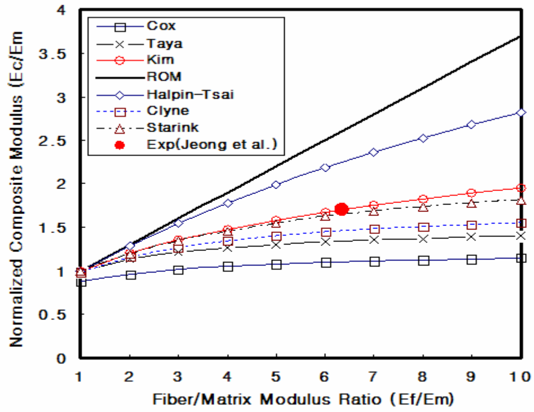
Fig. 8. Predicted and measured composite/matrix Young's modulus ratio as a function of fiber aspect ratio for V_f=30% in SiC-Al composites.

model underestimate from experimental data. However, it can be seen that the Clyne model and the Starink model are below experimental data for the very small fiber aspect ratio though they give good agreement with the Kim model as the fiber aspect ratio increases.

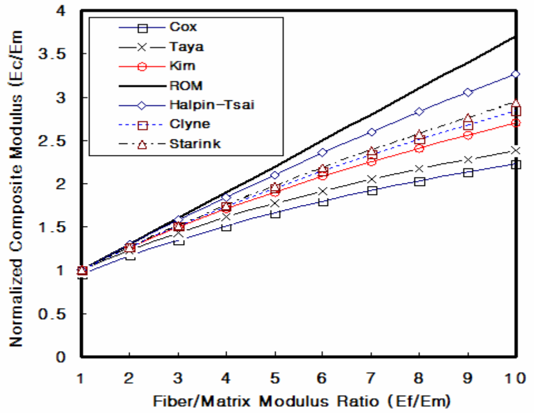
As can be seen in Fig. 8, the Kim model very closely matches the experimental data for the small fiber aspect ratio regime and fits the Clyne model and the Starink model with higher fiber aspect ratio regime. The error percentage of the Halpin-Tsai model indicating the upper bound except ROM model is 35.2%, that of the present model is 0.3%, and that of the Cox model indicating the lower bound is 42.3%. Presumably, all seven models converge to the ROM model as the fiber aspect ratios increases.

Fig. 9 shows the predicted value of composite/matrix Young's modulus ratio as a function of fiber/matrix Young's modulus ratio for V_f =30% in SiC-Al composites. Two cases of the fiber aspect ratio (s=3 and s=8) are depicted in Figs. 9 (a) and 9 (b), respectively. As can be seen in Fig. 9 (a), the Starink model and the Kim model give a good agreement with experimental data in the case of s=3.

Hence, three groups are found: (1) ROM model and Halpin-Tsai model: overestimated group, (2) Cox model, Taya model, and Clyne model: underestimated group, (3) Starink model and Kim model: moderate group. In the meantime, Fig. 9 (b) indicates that the discrepancy of each model becomes smaller for a larger fiber aspect ratio. However, three groups are found similarly. The error percentage of the Halpin-Tsai model indicating the upper bound except



(a) s=3



(b) s=8

Fig. 9. Prediction of composite/matrix Young's modulus ratio as a function of fiber/matrix Young's modulus ratio for V_f=30% in SiC-Al composites.

ROM model is 31.6%, that of the present model is 0.4%, and that of the Cox model indicating the lower bound is 45.4%.

4. Conclusions

A closed form solution of the shear lag theory has been derived and evaluated for the prediction of elastic modulus in short fiber reinforced discontinuous composite materials. The accuracy and relative simplicity of the present model have been exploited to derive an analytical model for the stress transfer in a composite and have also been compared with other theories. The predictions of the present model to predict the composite Young's modulus are fairly consistent with the measurements of SiC-Al MMCs. Our conclusions are as follows:

(1) For the effects of fiber volume fraction, the error percentage of the Halpin-Tsai model indicating the upper bound except ROM model is 36.3%, that of the present model is 4.9%, and that of the Cox model indicating the lower bound is 51.8%.
(2) For the effects of the fiber aspect ratio, the error percentage of the Halpin-Tsai model indicating the upper bound except ROM model is 35.2%, that of the present model is 0.3%, and that of the Cox model indicating the lower bound is 42.3%.
(3) For the effects of the fiber/matrix modulus ratio, the error percentage of the Halpin-Tsai model indicating the upper bound except ROM model is 31.6%, that of the present model is 0.4%, and that of the Cox model indicating the lower bound is 45.4%.

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